Construction of a Generic Supply Chain Simulation Model

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Abstract: Investigations into Supply Chain mechanics are now coming under closer scrutiny than ever before as industry professionals look for ways to reduce the amount of capital tied up in warehouses and on shop floors. By studying and capturing the dynamics of this challenging issue, BHP Services' information technology capability has constructed a model capable of quickly and easily simulating complex supply chains. The warehouses or factories at each link of the supply chain each have their own ordering policy and react individually to their customers' requirements. The customers generate requirements individually based on historical order patterns, random distributions, or a combination of both.

1 GLOSSARY

The following symbols are used throughout this paper to represent the following terms:

μ	= Mean
σ	= Standard Deviation
L	= Lead Time
D	= Lead Time Demand
β	= Aim Customer Service Level
I	= Inventory On Hand
U	= Unfulfilled Replenishment Order
C	= Unfulfilled Customer Order
Q	= Economic Order Quantity
O	= Demand Order Quantity
g	= Expected Shortage
R	= Reorder point
RQ	= Reorder quantity (for fixed periods)

2 BACKGROUND

1.1 History

BHP IT was asked to construct a flexible Supply Chain Simulation Model capable of quickly and easily creating supply chains of whatever complexity was required.

The aims of constructing the model were twofold. Firstly, the model is to be used as a tool to enable 'before and after' scenarios of a customer's business. Secondly, the model may be used as an investigative tool to examine the effectiveness of inventory control policies, and possible adverse interactions between different (or indeed similar) policies in large complex supply chains.

The policies discussed in this paper represent a subset of the policies represented in the model. The policies were chosen in part because they allow the approach of setting a pre-defined Customer Service Level to control the expected shortage per cycle.

1.2 Model Framework

The framework of the model is constant regardless of the actual inventory policy chosen. The following assumptions were made in the model framework:

- Customer orders are placed in the system at regular intervals, though some of the orders may be of size 0.
- The customer order amount is normally distributed about either a fixed mean, or a mean read in from a history file. In this study the order size distribution is N(20,2).
- The lead time between the placement of a replenishment order and its fulfillment may be drawn from a random distribution or fixed. In this study the lead time of 2 units has been used.
- Customer orders are satisfied immediately from stock;
- Where only part of an order may be satisfied from stock, that part will be satisfied immediately and the remainder of the order will be back-logged;
- During shortages customer orders are back-logged, not lost;
- Raw materials may be either 'manufactured' at a constant rate or be in infinite supply. In this study they are always available in infinite supply.

1.3 Policies Chosen

The policies implemented in the model were driven by the aim Customer Service Level (CSL), where the CSL is defined as the proportion of incoming orders that can be satisfied from stock on hand. The Customer Service Level then determines the expected shortage per cycle. Thus, according to Brown[1977].

$$\beta = 1 - \frac{\mu_s}{Q} \tag{1}$$

1.3.1 Periodic Review

This policy is appropriate where the supplier of goods operates on a fixed cycle – eg a rolling cycle at a steel mill.

From Classical Inventory Control Theory, the reorder point R which is the quantity of stock needed to cover the lead time demand plus a safety stock controlled by parameter K can be written as

$$R = \mu_D + K\sigma_D \tag{2}$$

where μ_D and σ_D are the mean and standard deviation of lead time demand. According to Fortuin [1980] K, for the logistic distribution, is defined as:

$$K = -\left(\frac{\sqrt{3}}{\Pi}\right) \times LN\left(EXP\left(\frac{\mu_O \Pi}{\sigma_D \sqrt{3}} \times (1-\beta)\right) - 1\right) \tag{3}$$

where the lead time is defined as the period between the placement of the current replenishment order, and the receipt of the next replenishment order – ie the period over which the current replenishment order will be used to fill customer orders.

Thus, the actual quantity to be ordered is

$$RQ = \mu_D + K\sigma_D - (I - U - C) \tag{4}$$

1.3.2 Continuous Review: Logistic

This policy is based on the same principles as the Periodic Review model above, but with inventory being checked against the calculated reorder point after every customer order. A replenishment order is then sent to the supplier if appropriate.

The main differences from the Periodic Review approach are:

- Continuous Review looks at the Economic Order Quantity (Q) rather than the Forecast Demand or average demand per period (μ_O)
- Lead Time refers to the time between a replenishment order being placed and the stock being received, rather than this time plus the period length which is the case for Periodic Review.

$$K = -\left(\frac{\sqrt{3}}{\Pi}\right) \times LN\left(EXP\left(\frac{Q\Pi}{\sigma_D \sqrt{3}} \times (1-\beta)\right) - 1\right) \tag{5}$$

Reorder if :

$$I - U - C \leq \mu_D + K\sigma_D \tag{6}$$

1.3.3 Continuous Review: Gamma Based

In the Continuous Review Model, when experimenting with shorter lead times, the assumption of symmetrically distributed demand (eg Normal or Logistic) would often be doubtful. The advantage of the Gamma distribution is that it can represent distributions with varying degrees of skewness.

Snyder [1984] showed how the Gamma distribution, which is defined for positive values only, can be modelled by one parameter instead of the usual two.

Still using (1) for the target service level, Snyder's method calculates the reorder point directly without the intermediate step of calculating the safety stock.

1.4 Software Chosen

The software we selected for this project was Arena (Professional Edition) from Simulation Modeling Services. This was selected for the following reasons:

1.4.1 Familiarity

BHP IT has been using Arena, and its predecessors Siman and Cinema, for about 15 years now. In that time we have developed a sound understanding of the software, its strengths, and its weaknesses.

1.4.1 Flexibility

The goal of the project was to provide a tool which could accurately reflect not only the inventory policies chosen at time of inception, but also to be able to have its capabilities enhanced with a minimal amount of coding.

This was necessary primarily as in the first phase of a consultation, the tool is to be used to model the customer's business in an 'as-is' mode.

Also, it was essential that the software chosen be able to be manipulated at a highly detailed level. This is obvious if the software is to be used as a research tool as it is necessary to not only to be able to enter new formulae and gather new data (if necessary), but also to be able to 'tweak' the actual logic of the model. An example of this 'tweaking' is studied in section 1.5.1

1.4.2 Ease of use of final model

With the intended users of the software being our Supply Chain/ Logistics Group, a supply chain model had to be able to be constructed and tailored to a customer's operations in a short space of time by people unfamiliar with simulation tools or theory. This was achieved through the use of the 'template' facility in Arena Professional Edition.

The use of templates enables to simply put blocks representing such standard elements as Customers, Dealers, Warehouses, and Suppliers together to model the customer's supply chain. The appropriate policies are chosen from the set currently existing within the blocks, the appropriate parameters are entered, and the simulation is run for a pre-specified number of replications, for a pre-specified though alterable run length.

1.5 Problems Encountered

During verification of the model, a number of inconsistencies were discovered in the model validation phase of the project. These observations both increased our knowledge of the Supply Chain/ Logistics domain considerably, but also led to a certain amount of insight into the mechanics of supply chain theory.

1.5.1 Discrete Supplier Lead Times

One potential hazard we encountered was linked to the inclusion of seemingly standard formulae without realising the impact of the specific settings of the simulation environment.

In this example, the lead times were set to a specific value with no variation, likewise customer orders arrived at the dealer module at an exact time every day. Whereas the formulae expect that both of these properties will be distributed uniformly throughout the day.

As an extreme example, take the case where the daily customer order arrives at 12.00 noon and the lead time is fixed at 2.0001 days. If a replenishment order is generated by today's customer order it will not be available to fulfill a customer order in two days time – as it will arrive .0001 days too late.

In this example, the lead time is effectively three days, but is recorded as 2.0001 days. The observable outcome of this problem is that achieved Customer Service Levels are consistently lower than the aim.

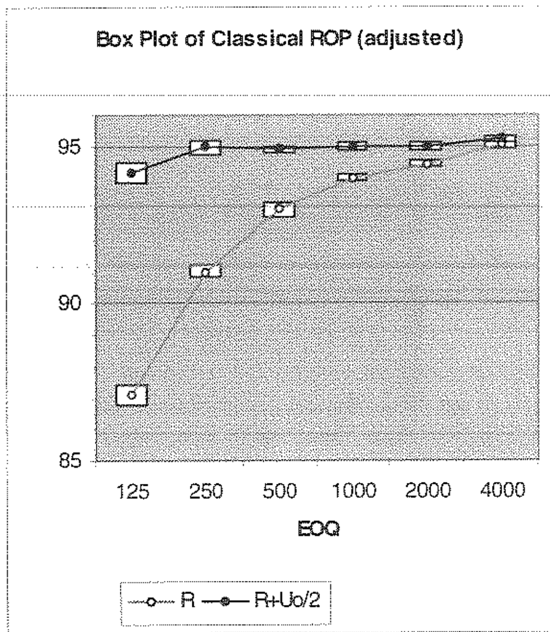
1.5.2 μ_O Greater Than One

Repeated use of the model with differing order patterns and quantities highlighted discrepancies between the input 'aim' CSL and the observed CSL.

Figure 1 displays the observed trend of results using the classical method with the logistic distribution (R). It was clear from the plot that as the EOQ becomes very large, the CSL achieved by the method approaches the aim CSL.

According to Williams [1982] 'the actual stock level when reordering takes place will not be R but some level below R, as one order will take the stock level from above R to below R'. Whilst this observation was made in reference to lumpy demand it is obvious that it is true in all cases, but becomes more noticeable as μ_O becomes large.

Figure 1



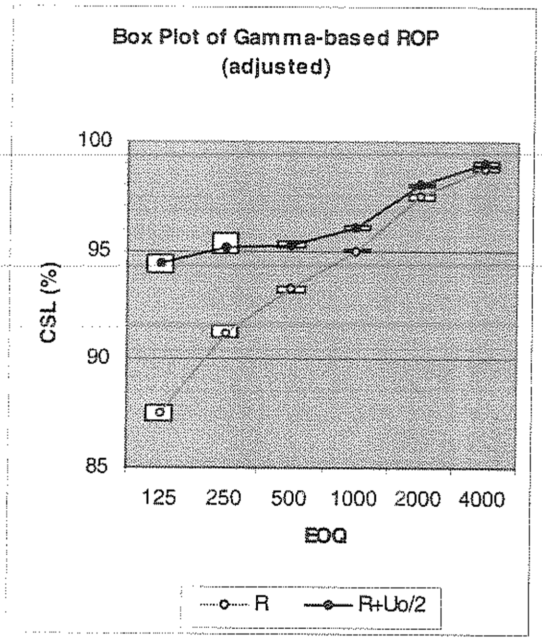
When one considers that the average stock level when a reorder has been triggered will not be R but somewhere between R and $R-\mu_O$, with the average stock level upon triggering a reorder being $R-\mu_O/2$. We see that in this case the expected lowest inventory level in a cycle is not equal to $R - L\mu_O$, but rather

$$E(Min(I)) = R - \left(L + \tfrac{1}{2}\right)\mu_O \qquad (5)$$

As a result, simply adding $\mu_O/2$ to the reorder point equations above would appear to give a much more accurate result as seen in Figure 1 ($R + \mu_O/2$).

Figure 2 displays the results for the Gamma based distribution. As can be seen, the addition of the $\mu_O/2$ term here has resulted in a similar increase while the EOQ is less than 1000, but a rising tail as the EOQ exceeds 1000. This is because the Gamma based policy does not allow for a negative R and as such the CSL continues to increase beyond 95% as Q increases. However even here one can see the marked improvement in results in the cases Q=125 (87.5% to 94.5%), Q=250 (92.2% to 95.1%), and Q=500 (93.2% to 95.2%)

Figure 2



3 Conclusions and Recommendations

The addition of the term $\mu_O/2$ to the calculated reorder point R for the classical method and the gamma-based methods discussed in this paper dramatically improves the achieved Customer Service Levels under the conditions mentioned above.

4 Acknowledgements

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3 References

Brown, R. G., *Materials Management Systems*, Wiley, New York, 1977.

Fortuin, L., Five Popular Probability Density Functions: A Comparison in the Field of Stock-Control Methods, *J. Opl. Res. Soc.*, (31), 937-942, 1980

Snyder, S., Inventory Control with the Gamma Probability Distribution, *Eur. Jnl. Opl. Res.*, (17), 373-381, 1984

Williams, T.M., Reorder Levels for Lumpy Demand, *J. Opl. Res. Soc., (33), 185-189, 1982*

Williams, T.M., Tables of Stock-Outs with Lumpy Demand, *J. Opl. Res. Soc., (34), 431-435, 1982*